

A Model of Baryon States

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I. INTRODUCTION

Some dynamical properties of the SU_3 symmetry model of strong interactions (1-13) have been considered by Cutkosky, Kalckar, and Tarjanne (14). It was possible to obtain a reasonably satisfactory description of the P -wave baryon-meson resonances, except that in both the triplet and the R -invariant (3, 6, 15) octet versions of the model it did appear that too many resonances would be predicted.

In an extension of the model (16) the mesons and baryons were themselves treated as bound states, in accordance with current ideas (17, 18). Only the triplet version of the SU_3 symmetry model was considered, although it proved to be possible to generalize the discussion to the group SU_n (the generalized Sakata model). A mechanism was suggested whereby the particles could be formed through the action of long range forces between the various particles of relatively low mass. It was assumed that the effects of the unknown short range forces could be simulated by cutting off the singular short range effects associated with the long range forces. It will be noted that if the baryons are to be formed as bound states of baryons with vector or pseudoscalar mesons, the lowest state must be $P_{1/2}$, not $S_{1/2}$. It was suggested that this could arise from the strong spinorbit force associated with vector meson exchange. Low-lying scalar continuum boson states (the one-dimensional multiplet is expected to be the most important) will also contribute to the proper ordering of the levels. Chew (19) has subsequently pointed out that exchange of baryonic states, especially the $P_{3/2}$ excited state, can play an important role.

In this paper we will apply the self-consistent bound state approach to the octet model. We follow the notation of ref. 14. We will not discuss the boson states, except to say that these are expected to be quite similar to the boson states of the triplet model, although the $B + \bar{B}$ components of these states will be more important. The strongest boson-boson forces are expected to occur in the one and eight dimensional multiplets. In particular, we assume here that the important states are vector and pseudoscalar states in the (1, 1) multiplet, and scalar states in the (0, 0) multiplet. Other (0, 0) states could easily be included.

The octet model has the property that the B - Π coupling constants and the "electric" and "magnetic" B - V coupling constants have ratios which are not completely fixed by the SU_3 symmetry—in each case there is an extra "mixing parameter" (3). Since the octet model allows the possibility of an additional symmetry operation R —the reflection of charge and hypercharge—Gell-Mann suggested that this arbitrariness might be removed by postulating invariance under R (3). It will be shown here that within the framework of our dynamical model, R -invariance is not compatible with the existence of only eight baryon states.¹ On the other hand, in this dynamical model it is not necessary to invoke an extra symmetry principle in order to fix the ratios of the coupling constants, because these are determined by the self-consistency requirement.² The self-consistency of the mixing parameters will be discussed in the next section, within the approximation of keeping only a few selected graphs.

In Section III we will estimate the mass differences within the baryon multiplet.

In the remainder of the paper we will examine the predictions our model gives for the properties of the $P_{3/2}$, $D_{3/2}$, and more highly excited states, and show that these agree with the limited experimental data that is currently available. Most of the group theoretical calculations we shall quote have been reported elsewhere (14, 21), but are summarized in the Appendix for easy reference.

In a dynamical model such as that considered here one has the possibility of making precise calculations of the predicted ratios of coupling constants and of the widths and relative positions of the resonances. In the present paper, however, we attempt to draw only such qualitative conclusions from the dynamical equations as do not require elaborate computations. The exploratory calculations reported here must eventually be supplemented by a more detailed analysis. This further study of the model is encouraged by the good agreement between the empirical observations and our exploratory calculation.

II. SELF-CONSISTENCY OF THE BARYON STATES

A. DISCUSSION OF VERTICES AND GRAPHS

The meson-baryon couplings

$$\bar{B}^i C_{ij}^k B^j (\Pi_k \text{ or } V_k)$$

are linear combinations of two terms (3).

¹ There is also some empirical evidence against R -invariance, which has been discussed in refs. 6, 7, and 20.

² One might ask at this point whether all of the coupling constant ratios implied by such symmetries as SU_3 or even isospin (SU_2) invariance, as well as the particular way in which the symmetries are broken, are a result of the requirement of self-consistency of the dynamical equations, or whether these special symmetries play the more fundamental role of distinguishing among many conceivable theories. Since the SU_3 model is, in fact, self-consistent at the level of approximation considered here, this question does not need to be considered further in the present paper.

$$C_{ij}^k = AF_{ij}^k + BD_{ij}^k, \quad (1)$$

where the relative values of A and B may depend on the type of meson and in the case of the vector meson, on the type of coupling. When one focuses one's attention on the π - N coupling constant $g_{\pi N}$, it is convenient to write (1) in Gell-Mann's form (3)

$$C_{ij}^k = (2/3)^{1/2} g_{\pi N} \{ \alpha D_{ij}^k + (1 - \alpha) F_{ij}^k \}. \quad (2)$$

The parametrization which is most convenient for dynamical calculations is

$$C_{ij}^k = g \{ (18)^{-1/2} \sin \theta F_{ij}^k + (10)^{-1/2} \cos \theta D_{ij}^k \}. \quad (3)$$

The relation between (2) and (3) is given by $\alpha = (1 + 1/3 \sqrt{5} \tan \theta)^{-1}$. We shall denote by θ the ΠB mixing angle and by φ and φ' the Dirac and Pauli VB mixing angles.

This arbitrariness is not present in the boson couplings. The Bose statistics ensure that the $\Pi^2 V$ coupling has the F form, and the ΠV^2 coupling has the D form. The V^3 vertex must be symmetrical under the interchange of every pair, which leads to the following coupling:

$$G_{V1} F_{ij}^k \{ (V_1^i \cdot V_2^j) V_3^k \cdot (P_1 - P_2) + (V_3^i \cdot V_2^j) V_1^k \cdot (P_3 - P_2) \\ + (V_1^i \cdot V_3^j) V_2^k \cdot (P_1 - P_3) \} + G_{V2} F_{ij}^k V_1^i \cdot P_2 V_2^j \cdot P_3 V_3^k \cdot P_1. \quad (4)$$

In the expression (4) the P_i are the momenta of the three lines, considered positive when directed into the vertex, with $\sum P_i = 0$. The couplings to the scalar states are:

$$G_{A\Pi} \Pi^i \Pi_i, \quad G_{AV} A V^i V_i, \quad \text{and} \quad G_{AA} A^3.$$

The boson vertices therefore involve 7 coupling constants. These coupling constants are, in principle, to be determined from the self-consistency of the boson states, but are not yet known to us, and we must therefore be content with a qualitative discussion of the baryon problem.

The one particle exchange graphs which contribute to the baryon structure are shown in Fig. 1. It is sufficient for us to treat the vector and scalar continuum states as though they were particles with a discrete mass. In most of our discussion we will neglect the baryon exchange graphs $B1-6$. At the end of this section we will discuss briefly these effects. The $D_{3/2}$ state of the baryons seems to be on nearly the same trajectory as the $P_{1/2}$ and $F_{5/2}$ baryon states, which suggests that the greater part of the binding potential may come from the non-exchange graphs.

We first examine the contribution to the baryon-meson potentials in the $(1, 1)$ representation. Since this representation occurs twice in the decomposition of the direct product $(1, 1) \otimes (1, 1)$, once in a symmetric (S) and once in an anti-symmetric (A) combination, each component of the potential will be a 2×2 matrix. (By potential, we mean more precisely the Born approximation to the

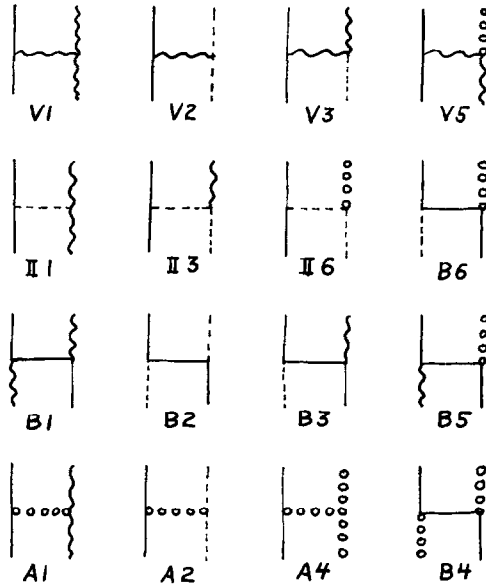


FIG. 1. One particle exchange graphs

scattering amplitude F , which is similar to the function N of the N/D method (22, 23).)

B. THE R INVARIANCE CATASTROPHE

If in graphs $V1$ and $V2$ there is a matrix F^i at each vertex, the potential depends only on the $(1, 1)$ character of the multiplets and not on their symmetry. The "bootstrap" model determines the sign of the $\bar{B}BV$ coupling constant, so the BV potential can be forced to be attractive. The sign of the $B\Pi$ potential then depends on the relative sign of the boson coupling constants. Let us assume that in the bootstrap model of the Π , there is an attractive $V\Pi$ force arising from V exchange; then the signs must be such that the $B\Pi$ force will also be attractive. The graphs $V3$ and $\Pi3$ have one F vertex and one D vertex, so that the $S(A)B\Pi$ states are coupled to the $A(S)BV$ states, with the same strength in both cases. So from these four graphs, we would infer the existence of another baryon octet (B'), which would have a behavior under R which was opposite to that of the original octet.

Graph $\Pi1$ has two D vertices, so this term in the potential has opposite signs in the S and A states. Let us assume the signs are such that it gives an attraction in the A multiplet. (Actually, the spin dependence of $\Pi1$ is different from that of $V1$; we will assume that we can argue on the basis of the expectation value of the potential in the baryon state.) Also note that the $B + A$ state is only coupled

to the A combination of $B + V$, and to the S combination of $B + \Pi$. If all of the coupling constants are approximately of the same magnitude, we can estimate that the average potential in the B' state will be at least half that of the B state. Now, for deeply bound states, the binding energy is much less sensitive to the strength of the potential than it is for weakly bound states. It is to be expected that halving the effective potential would still lead to a bound $B + V$ state.³

If a bound or resonant B' state arises, then the self-consistency calculations must be redone with this state included. We must introduce vertices with two B' lines—these have the same form as vertices with two B lines—and also vertices with one B and one B' . The latter vertices use the D coupling for V mesons, and the F coupling for Π mesons. Each solid line in the graphs of Fig. 1 must then be allowed to represent either a B or a B' . It is not hard to see that if all the B' coupling constants are assumed to be equal to the corresponding B coupling constants, and if the B and B' masses are assumed to be equal, then the potentials in the B and B' states must be identical, so that this choice is self-consistent. The difference in the potentials in the A and S states now does not affect the $B - B'$ mass difference, it merely makes the $B(B')$ more likely to disassociate into a $B(B')$ plus a meson than into a $B'(B)$ plus a meson (or possibly vice-versa). We wish to argue that no other self consistent solution arises from the model.

We think in terms of the partial wave dispersion relations for eight coupled channels: $B + (V_i, \Pi, A)$ and $B' + (V_i, \Pi, A)$ (where i distinguishes the two V spin states). We keep the mass M_B fixed at its empirical value; the potential is a function of $M_{B'}$, the twelve baryon-meson coupling constants, and an over-all cutoff parameter Λ . The location of the B and B' poles then gives two relations among these parameters. The ratios of the coupling constants are given by the eigenfunctions of the two scattering amplitudes. The magnitudes of the coupling constants are determined from the residues of the two poles. Note that the four $B'B$ coupling constants seem to be determined twice—once from the B state, and once from the B' state. However, there are really only four independent equations (which may be derived from a dispersion relation for the vertex), even though in the partial wave method with an approximate potential the superfluous equations might not be satisfied exactly.

It is seen that there are fourteen nonlinear equations to be solved for fourteen parameters. A solution exists with $M_{B'} = M_B$; we consider it very unlikely that another self-consistent solution with $M_{B'} \neq M_B$ could exist, because of the fact that the potential as calculated under the extreme assumption that $M_{B'} = \infty$ leads already to $M_{B'} - M_B \lesssim M_B$. If additional cutoff parameters were used

³ This argument is based on the fact that the radius of the bound state is comparable to, or smaller than, the range of the binding potential, and can be formulated either in terms of ordinary potential theory or the partial wave dispersion relations.

to modify the potentials, self-consistency with $M_{B'} \gg M_B$ could be obtained. However, this would imply that in any R -invariant model with only eight baryons the structure of the baryons would depend in an essential way on short range phenomena which lay outside the scope of the model.

The existence of another octet with $M_{B'} - M_B \gtrsim 700$ Mev is, of course, not yet ruled out by experiment. Nevertheless, this energy is sufficiently high that we must give up either R invariance or the hope that the baryon structure involves primarily the long range forces among the light particles.

C. SELF-CONSISTENCY OF THE GENERALIZED MODEL

If R invariance is denied, the potentials from graphs $V1$, $V2$, and $\Pi3$ have the form

$$V = V_0 \begin{pmatrix} \sin \chi & \cos \chi \\ \cos \chi & \sin \chi \end{pmatrix}, \quad (5)$$

and those from $\Pi1$ and $V3$ have the form

$$V = V_0 \begin{pmatrix} -\frac{3}{5} \cos \chi & \sin \chi \\ \sin \chi & \cos \chi \end{pmatrix}, \quad (6)$$

where χ denotes any of the angles θ , φ , φ' . We write the coupling constants for the pseudoscalar, vector, and tensor interactions in the form ga , gb , and gb' , where the normalization is chosen as in Eq. (3), with the extra condition $a^2 + b^2 + b'^2 = 1$.

It is sufficient for our purpose here to proceed as though the vector and pseudoscalar mesons had the same mass, and all elements of the Born approximation scattering matrix had the same energy dependence, because this simplification affects primarily the relative values of a , b , and b' , which we are not able to calculate anyway, and does not greatly influence the mixing angles. The N/D equations then take an especially simple form, because all of the 6×6 matrices are simultaneously diagonalized when we diagonalize the Born matrix. One eigenfunction of the Born matrix is then to be

$$\psi = (a \cos \theta, a \sin \theta, b \cos \varphi, b \sin \varphi, b' \cos \varphi', b' \sin \varphi')$$

according to the self-consistency requirement.

The amplitude from graph $V2$ has the form

$$F_{V2} = g \begin{pmatrix} xb \sin \varphi + x'b' \sin \varphi' & xb \cos \varphi + x'b' \cos \varphi' \\ xb \cos \varphi + x'b' \cos \varphi' & xb \sin \varphi + x'b' \sin \varphi' \end{pmatrix}, \quad (7)$$

where x and x' are to be considered as numbers obtained by averaging in some

way over the energy. Likewise, the amplitude from graph $\Pi 3$ may be written as:

$$F_{\Pi 3} = g \begin{pmatrix} za \sin \theta & za \cos \theta & z'a \sin \theta & z'a \cos \theta \\ za \cos \theta & za \sin \theta & z'a \cos \theta & z'a \sin \theta \end{pmatrix}. \quad (8)$$

The equations for the first two components of ψ are

$$\begin{aligned} \lambda a \cos \theta &= aA \cos \theta + aB \sin \theta, \\ \lambda a \sin \theta &= aA \sin \theta + aB \cos \theta, \end{aligned} \quad (9)$$

where the eigenvalue λ is a measure of the total effective strength of the interaction, and

$$\begin{aligned} A &= b(z+x) \sin \varphi + b'(z'+x') \sin \varphi' \\ B &= b(z+x) \cos \varphi + b'(z'+x') \cos \varphi'. \end{aligned} \quad (10)$$

One solution of (9) is $a = 0$, which we reject because it is not consistent with the approximation of neglecting $V3$. If $a \neq 0$, we have:

$$A \tan \theta + B = A \tan \theta + B \tan^2 \theta. \quad (11)$$

A solution of this is $B = 0$, which may be achieved with $\cos \varphi = \cos \varphi' = 0$. If this is true, the other four equations (not written) are satisfied with $\theta = 0$. This is the R -invariant solution, which, while superficially self-consistent, leads to too strong an attraction in a state orthogonal to ψ . (There may also be other solutions, which we have not studied, with $B = 0$.)

If $B \neq 0$, we obtain from (11) that $\theta = \pm 45^\circ$, and

$$\lambda = 2^{1/2}b(z+x) \sin(\theta + \varphi) + 2^{1/2}b'(z'+x') \sin(\theta + \varphi'). \quad (12)$$

If we consider the remaining components of ψ , and just take into account graph $V1$, we find that a possible solution is $\varphi = \varphi' = \theta$. However, this solution is still not unique; other values of φ and φ' are also consistent with $\theta = \pm 45^\circ$. These other solutions depend on the values of the x 's, y 's, and z 's (where the y 's are the $V1$ amplitudes). When the other graphs are included, the values of the self-consistent angles will be modified, but we can not expect the nonuniqueness to disappear.

We do not have any general criterion for choosing among these different solutions, except that we require $\lambda > 0$, so that the potential is really attractive. We have already given an argument for believing that $V1$ and $V2$ are simultaneously attractive; also, it turns out that x and z have the same sign. We may, however, without loss of generality at this point, choose θ so that $0 < \alpha < 1$. This means that we choose to define, provisionally, the nucleons to be the

doublet with the larger coupling to pions, and the Ξ 's to be the doublet with the smaller pion coupling. In the next section we shall show that this leads to $M_N < M_\Xi$.

The different possible solutions have different values of λ . In order to achieve self-consistency, the cutoff Λ is adjusted (inversely with λ). In the future, it may be possible to go beyond the one particle approximation and calculate Λ approximately, thereby perhaps narrowing the choice of solutions. Even within the one particle exchange approximation, we must be sure that the other eigenvalues calculated with the same potential are not so large that other, unwanted, bound states would be generated; this was the way we ruled out the R -invariant solution. (These different eigensolutions of the linear problem with a given potential must not be confused with the different self-consistent solutions of the nonlinear bootstrap problem.)

Let us first look at the other $(1, 1)$ states, lumping together the vector and tensor couplings of the vector meson (so we can ignore the primed variables and parameters). We consider only graphs $V1$, $V2$, and $\Pi3$, and the solution $\theta = \varphi = 45^\circ$. We have a 4×4 matrix which has two zero eigenvalues, and two others given by

$$\lambda_1 = 2^{1/2}b(x + z), \quad \lambda_2 = 2^{1/2}b(y - z), \quad b = z[2z^2 + z(x - y)]^{-1/2}. \quad (13)$$

We expect x , y , and z to have a similar magnitude, or $\lambda_2 \ll \lambda_1$. While the actual problem is much more complicated than this, the above results suggest that it is reasonable to expect that a solution could be found in which there was only one strongly attracted $(1, 1)$ state.

We may note here that the $B + A$ state is automatically coupled directly only to the self-consistent $B + \Pi$ state through graphs $\Pi6$ and $B6$. Moreover, the strength of the coupling is independent of θ . The existence of two VB states makes the effect of graphs $V5$ and $B5$ a little more complicated. The potential from graphs $A1$ and $A2$ is independent of the multiplet, but is not so strong, since there is only one A particle, but eight Π 's and eight V 's. These arguments are essentially independent of the A spin and parity.

The strength of the forces in the other multiplets obtained through the reduction of $(1, 1) \otimes (1, 1)$ may be estimated from Table I, which lists the average potentials from graphs $V1-3$, $\Pi1$, $\Pi3$, and $B2$. We have assumed that the difference between φ and φ' can be neglected (or else that $b' \ll b$). For the $(1, 1)$ multiplet, the potential listed is the matrix element of the potential between normalized states having the self-consistent values of the mixing angles θ and φ . We have not taken specific values for these angles, but we do wish to assume, provisionally, that θ and φ do not differ greatly from 45° . The signs of x , y , and z are assumed to be positive. The parameter ξ is positive in the $P_{1/2}$ state (as in the static nucleon model, $\xi(P_{3/2}) \approx -2\xi(P_{1/2})$). The signs of η and ζ are not known because they depend on the sign of $g_{\Pi V}$.

TABLE I
POTENTIALS ARISING FROM VARIOUS GRAPHS^a

	V1	(yb)
(2, 2)	$-\frac{2}{3} \sin \varphi$	
(3, 0)	$-(2/5^{1/2}) \cos \varphi$	
(0, 3)	$+(2/5^{1/2}) \cos \varphi$	
(1, 1)	$\sin \varphi (1 + 2 \cos^2 \varphi)$	
(0, 0)	$2 \sin \varphi$	
	V2	(xb)
(2, 2)	$-\frac{2}{3} \sin \varphi$	
(3, 0)	$-(2/5^{1/2}) \cos \varphi$	
(0, 3)	$+(2/5^{1/2}) \cos \varphi$	
(1, 1)	$\sin \varphi + \cos \varphi \sin 2 \theta$	
(0, 0)	$2 \sin \varphi$	
	Π3	(za)
(2, 2)	$-\frac{2}{3} \sin \theta$	
(3, 0)	$-(2/5^{1/2}) \cos \theta$	
(0, 3)	$+(2/5^{1/2}) \cos \theta$	
(1, 1)	$\sin \varphi + \cos \varphi \sin 2 \theta$	
(0, 0)	$2 \sin \theta$	
	Π1	(ηa)
(2, 2)	$\frac{2}{5} \cos \theta$	
(3, 0)	$-\frac{4}{5} \cos \theta + (2/5^{1/2}) \sin \theta$	
(0, 3)	$-\frac{4}{5} \cos \theta - (2/5^{1/2}) \sin \theta$	
(1, 1)	$\cos \theta (\sin^2 \varphi - \frac{3}{5} \cos^2 \varphi) + \sin \theta \sin 2 \varphi$	
(0, 0)	$2 \cos \theta$	
	V3	(zb)
(2, 2)	$\frac{2}{5} \cos \varphi$	
(3, 0)	$-\frac{4}{5} \cos \varphi + (2/5^{1/2}) \sin \varphi$	
(0, 3)	$-\frac{4}{5} \cos \varphi - (2/5^{1/2}) \sin \varphi$	
(1, 1)	$\cos \theta (\sin^2 \varphi - \frac{3}{5} \cos^2 \varphi) + \sin \theta \sin 2 \varphi$	
(0, 0)	$2 \cos \varphi$	
	B2	(ξa ²)
(2, 2)	$-(\frac{3}{5} \cos^2 \theta + \frac{2}{3} \sin^2 \theta)$	
(3, 0)	$-(\frac{3}{5} \cos^2 \theta + (4/5^{1/2}) \sin \theta \cos \theta)$	
(0, 3)	$-\frac{4}{5} \cos^2 \theta + (4/5^{1/2}) \sin \theta \cos \theta$	
(1, 1)	$\frac{3}{5} \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta - \sin^4 \theta$	
(0, 0)	$2(\sin^2 \theta - \cos^2 \theta)$	

^a Each entry is to be multiplied by a factor (xb), (yb), etc, as indicated. A factor $3(2)^{-1/2}$ has been absorbed into the definitions of x, y, etc.

Table I shows that the forces in the (0, 0) state and the (1, 1) state have nearly the same strength. In the (0, 3) state the potential is perhaps about half as great, but the relative value is very sensitive to the sign of η and ζ . In both cases, the relative strength of the potentials also depends on θ and φ .

The baryon exchange graphs give a contribution to the potential which alternates in sign in states with successive values of j . That graph $B2$ is attractive in the $(1, 1)$ $P_{1/2}$ state is seen from Table I; the same is easily seen to be true of $B6$. Whatever the relative signs of the BBA and $BB\Pi$ couplings, graph $B4$ has a positive expectation. Graph $B5$ is more complicated, but the same is certainly true when the $B + A$ state has little energy. We shall not discuss $B1$ and $B3$ here.

It will be shown in Section V that when $10^\circ \lesssim \theta \lesssim 60^\circ$, the $B\Pi$ interactions are favorable for generating a $j = 3/2 (+)$ resonance (B^*) in the tenfold $(3, 0)$ multiplet. The $B\Pi$ potential arising from B^* exchange is obtained from the crossing matrix in Table II. This potential plays the dominant role in Chew's mechanism (19) for producing the B as a bound state. It is four times as strong in the $P_{1/2}$ state as in the $P_{3/2}$ state, in the approximation in which one neglects the Π mass. According to Table II, the B^* exchange potential is strongly repulsive in the $(0, 0)$ multiplet. There is an almost equally strong attraction in the $(1, 1)$ multiplet, in an eigenstate which corresponds to a mixing angle given by $\tan \theta = 5^{1/2}(1 + 6^{1/2})^{-1}$, or $\theta = 33^\circ$. In the other states, this potential is unimportant.

We conclude that the boson exchange graphs, and also all the baryon exchange graphs which can be easily studied, are strongly attractive in the $(1, 1)$ multiplet, in the $P_{1/2}$ and $F_{5/2}$ states. The baryonic potential is repulsive in the $D_{3/2}$ state. A detailed calculation would enable one to calculate the shapes of the even and odd Regge trajectories. Fortunately, a qualitative value for θ is obtained independently of such a detailed calculation, because both types of graphs lead to similar values.

It is interesting to observe that if $20^\circ < \theta < 50^\circ$, the $\bar{N}N\pi$ coupling constant is particularly large; in the virtual dissociation $N \rightarrow B + \Pi$, more than two thirds of the time the $B + \Pi$ combination is $N + \pi$, even if the mass differences are neglected. As a consequence, many of the discussions of $S = 0$ baryon states in which strange particles were disregarded remain good approximations in the SU_3 symmetry model.

III. THE BARYON MASS DIFFERENCES

Our model of the baryon mass differences is based on the observation that the lightest baryons are coupled most strongly to the lightest pseudoscalar mesons. We assume the mass differences of the pseudoscalar mesons are given, and do not try to explain them. We assume temporarily that all other sources of dissymmetry in the baryon problem, such as the coupling constant ratios and the vector meson masses, may be disregarded.

Gell-Mann (3) and Okubo (8) have given arguments for the validity of the following empirical mass relations:

$$\delta_0 \equiv M_K^2 - \frac{3}{4}M_\pi^2 - \frac{1}{4}M_\tau^2 = 0, \quad (14)$$

TABLE II
CROSSING MATRIX A , MULTIPLIED BY 120^a

	(0, 0)	(2, 2)	(0, 3)	(3, 0)	(1, 1)		
					A	S	Q
(0, 0)	15	15	-15	-15	-15	15	0
(2, 2)	405	21	27	27	135	81	0
(0, 3)	-150	10	30	30	0	60	$-30\sqrt{5}$
(3, 0)	-150	10	30	30	0	60	$+30\sqrt{5}$
A	-120	40	0	0	60	-60	0
S	120	24	48	48	-60	-36	0
Q	0	0	$-48\sqrt{5}$	$+48\sqrt{5}$	0	0	0

^a This gives the potential in the column multiplet arising from an exchanged row multiplet.

$$\Delta_0 \equiv \frac{1}{2}M_N^2 + \frac{1}{2}M_\Sigma^2 - \frac{3}{4}M_\Lambda^2 - \frac{1}{4}M_\Sigma^2 = 0. \quad (15)$$

In our model, (15) will be a consequence of assuming (14).

We denote by $(N\pi)$ a normalized $T = \frac{1}{2}$ state containing one nucleon and one pion, and use a similar notation for other $B + \Pi$ states. The explicit decomposition of $(1, 1) \otimes (1, 1)$ has been given by many authors so we only need to quote the results here (21, 24, 25, 26):

$$\begin{aligned} \psi_N = & (\frac{1}{2} \sin \theta + \frac{3}{2}5^{-1/2} \cos \theta)(N\pi) + (\frac{1}{2} \sin \theta - \frac{1}{2}5^{-1/2} \cos \theta)(N\eta) \\ & - (\frac{1}{2} \sin \theta + \frac{1}{2}5^{-1/2} \cos \theta)(\Lambda K) - (\frac{1}{2} \sin \theta - \frac{3}{2}5^{-1/2} \cos \theta)(\Sigma K), \end{aligned} \quad (16a)$$

$$\begin{aligned} \psi_\Sigma = & (\frac{1}{2} \sin \theta + \frac{3}{2}5^{-1/2} \cos \theta)\Sigma\bar{K} + (\frac{1}{2} \sin \theta - \frac{1}{2}5^{-2/3} \cos \theta)(\Lambda\bar{K}) \\ & - (\frac{1}{2} \sin \theta + \frac{1}{2}5^{-1/2} \cos \theta)(\Xi\eta) - (\frac{1}{2} \sin \theta - \frac{3}{2}5^{-1/2} \cos \theta)(\Xi\pi) \end{aligned} \quad (16b)$$

$$\begin{aligned} \psi_\Lambda = & (2^{-1/2} \sin \theta + 10^{-1/2} \cos \theta)(N\bar{K}) + (\frac{3}{5})^{1/2} \cos \theta(\Sigma\pi) \\ & - 5^{-1/2} \cos \theta(\Lambda\eta) - (2^{-1/2} \sin \theta - 10^{-1/2} \cos \theta)(\Xi K) \end{aligned} \quad (16c)$$

$$\begin{aligned} \psi_\Sigma = & (6^{-1/2} \sin \theta - (\frac{3}{10})^{1/2} \cos \theta)(N\bar{K}) + 5^{-1/2} \cos \theta(\Sigma\eta) \\ & - (\frac{2}{3})^{1/2} \sin \theta(\Sigma\pi) + 5^{-1/2} \cos \theta(\Xi\pi) \\ & - (6^{-1/2} \sin \theta + (\frac{3}{10})^{1/2} \cos \theta)(\Xi K). \end{aligned} \quad (16d)$$

The position W^2 of a zero of D (in the N/D method) will be a function of the masses that were assumed in calculating D from its dispersion relation. We write:

$$\beta = dW^2/dM_B^2, \quad \gamma = dW^2/dM_\pi^2. \quad (17)$$

These are calculated for a one-channel problem, that is, on the assumption that only one given B and one given Π enter into the state. In our problem we have to calculate the expectation values in the states (16), so that, for example:

$$\begin{aligned}
 dM_N^2 = & \beta[(\frac{1}{2} + 5^{-1/2} \sin \theta \cos \theta) dM_N^2 \\
 & + (\frac{1}{4} \sin^2 \theta + \frac{1}{2}(5)^{-1/2} \sin \theta \cos \theta + (\frac{1}{20}) \cos^2 \theta) dM_\Lambda^2 \\
 & + (\frac{1}{4} \sin^2 \theta - \frac{3}{2}(5)^{-1/2} \sin \theta \cos \theta + (\frac{9}{20}) \cos^2 \theta) dM_\Sigma^2] \\
 & + \gamma[(\frac{1}{2} - 5^{-1/2} \sin \theta \cos \theta) dM_K^2 \\
 & + (\frac{1}{4} \sin^2 \theta - \frac{1}{2}(5)^{-1/2} \sin \theta \cos \theta + (\frac{1}{20}) \cos^2 \theta) dM_\eta^2 \\
 & + (\frac{1}{4} \sin^2 \theta + \frac{3}{2}(5)^{-1/2} \sin \theta \cos \theta + (\frac{9}{20}) \cos^2 \theta) dM_\pi^2].
 \end{aligned} \tag{18}$$

We express the combinations Δ_0 and

$$\Delta_1 = M_\Lambda^2 - M_\Sigma^2, \quad \Delta_2 = M_\Xi^2 - M_N^2, \tag{19}$$

in terms of δ_0 and $\delta_1 = M_\eta^2 - M_\pi^2$. We find that:

$$\Delta_0 = a_1(\beta\Delta_0 + \gamma\delta_0) \tag{20a}$$

$$\Delta_1 = -\beta(2a_1\Delta_0 + \frac{3}{2}a_1\Delta_1 + a_2\Delta_2) - \gamma(2a_1\delta_0 + \frac{3}{2}a_1\delta_1) \tag{20b}$$

$$\Delta_2 = \beta(-a_2\Delta_0 - \frac{5}{4}a_2\Delta_1 + \frac{1}{2}\Delta_2) + \gamma a_2(\delta_0 + \frac{5}{4}\delta_1), \tag{20c}$$

where

$$a_1 = \frac{1}{5} \cos^2 \theta - \frac{1}{3} \sin^2 \theta$$

$$a_2 = 2(5)^{-1/2} \sin \theta \cos \theta.$$

Since $\delta_0 = 0$, (20a) gives $\Delta_0 = 0$, and we also find:

$$\Delta_1 = -\gamma\delta_1 R_1/S, \quad \Delta_2 = \gamma\delta_1 R_2/S, \tag{21}$$

where

$$R_1 = \frac{3}{2}a_1 + \frac{1}{4}\beta(5a_2^2 - 3a_1)$$

$$R_2 = \frac{5}{4}a_2(1 + 3\beta a_1) \tag{22}$$

$$S = 1 - \frac{1}{2}\beta(1 - 3a_1) - \frac{1}{4}\beta^2(5a_2^2 + 3a_1).$$

In view of the crudity of our model, it is not worth while to calculate β and γ from an exact solution of the N/D equations, but we must at least try to estimate plausible values for them. We write the integral

$$D(s) = 1 - \int \rho(s)N(s') ds'/\pi(s' - s) \tag{23}$$

as

$$D(s) = 1 - A/(\Omega^2 - s), \quad (24)$$

where Ω^2 is an average value of s' in (23) (Ω is about $2M_B$ or $3M_B$), and

$$A = \int (\Omega^2 - s)\rho'N' ds'/\pi(s' - s), \quad (25)$$

which we assume to be nearly independent of s when s is near W^2 . We shall also use the approximations

$$d\Omega^2/dM^2 = 0 \quad (26a)$$

and

$$dA/dM^2 = A[d(\rho N)/dM^2](\rho N)^{-1} \quad (26b)$$

where ρ and N are evaluated at $s' = \Omega^2$, which leads, since $W^2 = \Omega^2 - A$, to:

$$\frac{dW^2}{dM^2} = -\left(\frac{\Omega^2 - W^2}{p^2} \frac{dp^2}{dM^2}\right) \left(\frac{p^2 d \log \rho N}{dp^2}\right). \quad (27)$$

In Eq. (27), p^2 is the barycentric momentum at $s = \Omega^2$, and

$$-\frac{\Omega^2 - W^2}{p^2} \frac{dp^2}{dM_B^2} = \frac{\Omega^2 - M_B^2}{(\Omega - M_\Pi)^2 - M_B^2} + \frac{\Omega^2 - M_B^2}{(\Omega + M_\Pi)^2 - M_B^2} \quad (28a)$$

$$-\frac{\Omega^2 - W^2}{p^2} \frac{dp^2}{dM_\Pi^2} = \frac{\Omega^2 - M_B^2}{(\Omega - M_B)^2 - M_\Pi^2} + \frac{\Omega^2 - M_B^2}{(\Omega + M_B)^2 - M_\Pi^2}. \quad (28b)$$

We assume that N resembles the Born approximation, or, more precisely, that $N \propto (M_v^2 + 2p^2)^{-1}$, which gives

$$p^2 d(\log N)/dp^2 \approx -2p^2/(M_v^2 + 2p^2) \approx -\frac{3}{4}. \quad (29)$$

It is very important that (29) does not overwhelm the term

$$p^2 d(\log \rho)/dp^2 = \frac{3}{2}. \quad (30)$$

Our determination of the sign of Δ_1 and Δ_2 therefore depends critically on the velocity dependence of the B - Π potential.

We should also include in γ a factor x ($x \gtrsim \frac{1}{2}$) to represent the fractional contribution of the $B + \Pi$ channel to the state of the B . It seems plausible that this factor should be omitted from β . When we collect our guesses about the factors in (27), we conclude that β and γ should be about unity, but there is in our estimate a considerable uncertainty. From the experimental values of the mass differences we obtain:

$$\Delta_1 = -0.63\delta_1 \quad (31)$$

$$\Delta_2 = +2.0\delta_1.$$

The correct signs of both Δ_1 and Δ_2 are reproduced by Eq. (21) for $5^\circ \lesssim \theta \lesssim 60^\circ$, for a large range of (positive) values of β and γ . The correct order of magnitude is also obtained for reasonable values of β and γ , although the ratio $r = -\Delta_2/\Delta_1$ turns out systematically to be about 2 or 3—empirically, $r = 5$. For example, at $\alpha = \frac{3}{4}$ ($\theta = 24.1^\circ$) we find, independently of β and γ , that $r = 2\frac{1}{2}$.

In the $j = \frac{3}{2}(-)$ octet the value of r is probably no more than unity (as yet, not all members of the octet have been found). It is likely that only the average value of Δ_2 should be given by our simple model, and that a large contribution is also obtained from the baryon exchange graphs. This might arise in the following way: the B^* exchange potential will be greatest for the states in which the exchanged B^* is lightest, that is, larger for $S = 0$ than for $S = -2$. This leads to a positive contribution to Δ_2 for the $P_{1/2}$ and $F_{5/2}$ states, and a negative contribution for the $D_{3/2}$ state.

Finally, we mention the effect of the vector meson mass differences. These may also give sizeable contributions to Δ_1 and Δ_2 . In particular, the failure of the G-O formula for the vector mesons reproduces itself in a failure of the formula for the baryons, unless one assumes with Sakurai (27) that this failure arises from interaction with an additional (0, 0) (singlet) vector meson.

IV. THE EXCITED OCTET STATES

The $j = \frac{3}{2}(-)$ and $j = \frac{5}{2}(+)$ octet baryon states can decay into a member of $j = \frac{1}{2}(+)$ octet and a pseudoscalar meson. If the SU_3 symmetry were exact, the matrix elements $M(j' = \frac{1}{2}, j)$ for these transitions would be given by linear combinations of the F and D types of terms, as in Eq. (1) for $j = \frac{1}{2}$. The mixing angles for these matrix elements are independent of j , provided we neglect the differences in the angular momentum dependences of the various graphs; in particular, the baryon exchange graphs would have to be omitted. Alternatively, the B^* potential will not generate differences between the mixing angles θ if θ is close to that of the eigenfunction of the B^* exchange potential ($\theta = 33^\circ$).

The nondegeneracy of the members of a supermultiplet leads to dynamical effects which completely change the matrix elements for decay. These effects arise from the dependence of the phase space factors and the centrifugal barrier penetration factors on the masses of the particles. We estimate their influence from the formula

$$\Gamma = \gamma p^{2l+1}/(1 + a^2 p^2)^{2l}, \quad (32)$$

where p is the momentum of the decay particles, Γ is the observed partial width, and γ is a reduced width. We expect the ratios of the different γ 's to exhibit the SU_3 symmetry. The parameter a is a measure of the radius of the state; we shall use $a^{-1} = 1$ Bev. (This parameter can be changed by 50% without materially

affecting our results.⁴) We note that in (32) with $l = 1$ (and $a = 0$), γ corresponds to the square of a pseudoscalar coupling constant.

The form of Eq. (32) is suggested by the Schrödinger and Bethe-Salpeter amplitudes. The widths Γ could be calculated more precisely from the N/D method. We would assume that the nearby left hand cuts could be calculated from the SU_3 -symmetric model, and, if necessary, adjust the distant singularities to get the resonance at the correct energy W . The width of a narrow resonance is given by

$$\frac{1}{2}\Gamma s_0^{1/2} d(\text{Re } D(s))/ds = \rho(s)N(s) \quad (33)$$

(evaluated at $s_0 = W^2$), where $\rho(s)$ provides the factor p^{2l+1} in (36). Relative values of N are given by SU_3 symmetry, except that we introduce the denominator in (32) to represent the fact that ρN does not blow up at $p \rightarrow \infty$. It is sufficient for our purposes to assume the parameter a is constant within a super multiplet and make a reasonable guess as to its value, instead of trying to calculate a from (33).

The states N' (1512 Mev) and Λ' (1520 Mev) are interpreted as members of the $j = \frac{3}{2}(-)$ octet. We use the following values for the partial widths:

$$\Gamma(N' \rightarrow N + \pi) = 110 \text{ Mev} \quad (28)$$

$$\left. \begin{aligned} \Gamma(\Lambda' \rightarrow \Sigma + \pi) &= 9 \text{ Mev} \\ \Gamma(\Lambda' \rightarrow N + K) &= 4\frac{1}{2} \text{ Mev} \end{aligned} \right\} \quad (29).$$

The uncertainty in these numbers is perhaps as much as 20%. We calculate from Eq. (32) (using 1 Bev as the unit) that

$$\begin{aligned} \gamma(N' \rightarrow N + \pi) &= 12.4 \\ \gamma(\Lambda' \rightarrow \Sigma + \pi) &= 9.1 \\ \gamma(\Lambda' \rightarrow N + K) &= 7.1 \end{aligned} \quad (34)$$

We wish to emphasize that the values above show directly that the interactions of K mesons and π mesons have a similar strength.

Predicted relative values of the γ 's are obtained by squaring the coefficients in Eq. (16 a-d), and are plotted in Fig. 2. These values are normalized to a unit total decay probability for the (hypothetical) case of perfect SU_3 symmetry. The ratio $\gamma(\Lambda' \rightarrow \Sigma + \pi)/\gamma(\Lambda' \rightarrow N + K)$ is quite sensitive to θ and agrees with the empirical ratio for $\theta = 28^\circ \pm 5^\circ$. The ratio $\gamma(N' \rightarrow N + \pi)/[\gamma(\Lambda' \rightarrow \Sigma + \pi) + \gamma(\Lambda' \rightarrow N + K)]$ is almost independent of θ in this range and is in excellent agreement with (34).

⁴ In ref. 14, the value $a = 0$ was used.

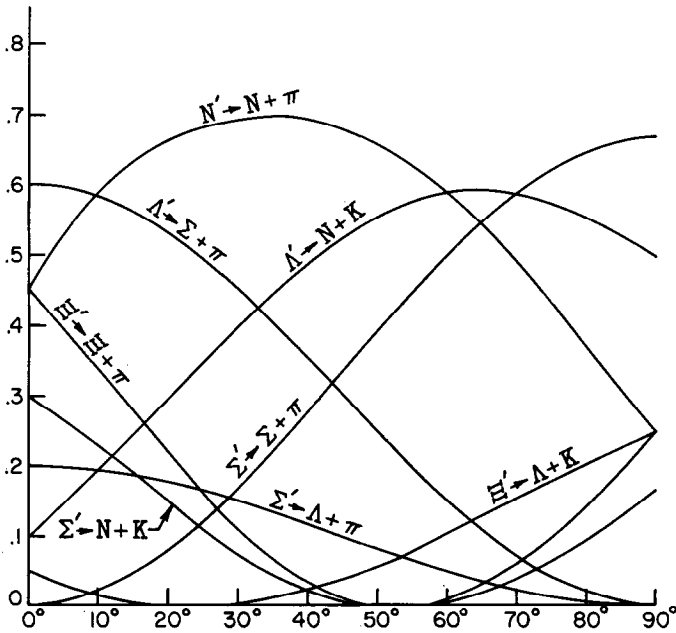


FIG. 2. Reduced decay widths for decay of the $D_{3/2}$ baryon isobars, normalized to unit total two particle width.

Since we have fit two ratios by adjusting the two parameters θ and a this is perhaps not a significant test of SU_3 symmetry, even though the values do turn out to be extremely reasonable. However, we can now predict the reduced widths of the two remaining components of the octet (Σ' and Ξ'). These widths are also plotted in Fig. 2. They turn out to be somewhat smaller than the widths of the N' and Λ' . It should be remembered that some theoretical uncertainty is introduced into the calculation of Γ through the use of Eq. (32).

A new resonance at 1650 Mev (37) has been identified as the Σ' by Glashow and Rosenfeld (38). The widths agree with those predicted above about as well as one ought to expect. Glashow and Rosenfeld, assuming $\Gamma(N' \rightarrow N + \pi) = 80$ Mev, obtain $a = 2.8 \text{ Bev}^{-1}$ and $\theta = 35^\circ$. However, it seems that intermediate values of a and θ give the best fit. (Glashow and Rosenfeld also define a differently.)

V. THE $P_{3/2}$ RESONANCES

The work of Chew and Low (30) as supplemented by the studies by Frautschi and Walecka (31) and by Bowcock, Cottingham, and Lurie (32), shows that ΠB scattering in $P_{3/2}$ states is dominated by graph B2. In ref. 14, it was shown that with an R invariant coupling, the attraction in the (2, 2) state was one-half

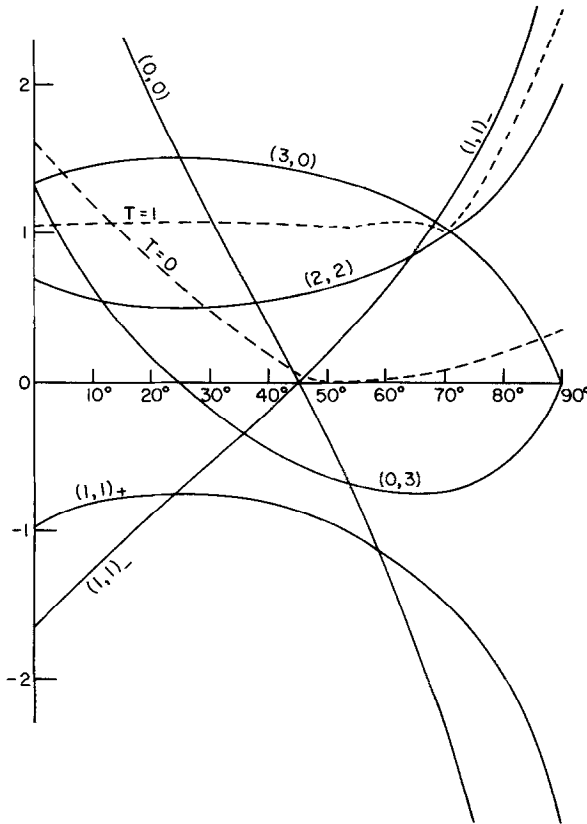


FIG. 3. The solid lines give the strength of the baryon pole term in B - Π scattering, versus the mixing angle θ , in various SU multiplets, relative to a unit pole strength in $T = \frac{3}{2}$ N - π scattering. The dashed lines give the effective pole strength in $S = 1$, $T = 0$ and $T = 1$, B - Π scattering, when the $(\Sigma\eta)$, $(\Lambda\eta)$, and (ΞK) channels are omitted.

that in the $(3, 0)$ and $(0, 3)$ states. When R invariance is given up, the relative potentials from $B2$ are as plotted in Fig. 3. For the values of θ which are consistent with those suggested in the earlier part of this paper ($15^\circ \lesssim \theta \lesssim 45^\circ$) the $(3, 0)$ potential is between two and three times as attractive as the $(2, 2)$ potential, and the $(0, 3)$ potential is either very weak or repulsive (39).⁵ That a $P_{3/2}$ resonance should occur in the $(3, 0)$ supermultiplet (if at all) is therefore an unambiguous prediction of SU_3 symmetry. This 10-fold supermultiplet evidently contains the following states: $N_{3/2}^*$, Y_1^* , $\Xi_{1/2}^*$, and Ω_0 ; the last is an $S = -3$ state which is expected to be stable under strong interactions.⁶

⁵ Gell-Mann has also calculated the $(3, 0)$ and $(2, 2)$ potentials (private communication).

⁶ Glashow and Sakurai (33) have suggested that a mysterious cosmic ray event observed by Eisenberg (34) might be interpreted as the weak decay $\Omega^- \rightarrow \Sigma^0 + K^-$, with $M_\Omega = 1690$ Mev.

Since the potential in the (2, 2) states is also attractive, we would expect that the mass differences could easily produce large admixtures of (2, 2) components into the states of the 10-fold supermultiplet. For example, the $T = \frac{3}{2}, S = 0$ members of the (3, 0) and (2, 2) supermultiplets are both mixtures of ($N\pi$) and (ΣK) states. In the original theory of the $N_{3/2}^*$ resonance, it was necessary to cut off the integral for $\text{Re } D(s)$ at such a place that the main contribution to the integral came about 600 Mev above the πN threshold (or $\Omega^2 \lesssim 3M_B^2$, in the notation of Eq. (28)). At this energy, the (ΣK) channel is just opening. Therefore, the $N_{3/2}^*$ is associated predominately with the ($N\pi$) channel, or in other words, with an almost equal mixture of the (3, 0) and (2, 2) states.

Cutkosky, Kalckar, and Tarjanne (14) suggested on the basis of these observations that in the $\text{Re}D$ integral for the Y_1^* state, it would be a reasonable approximation to omit the ($\Sigma\eta$) and (ΞK) channels, and assume the phase space was equal in the ($\Lambda\pi$), ($\Sigma\pi$), and ($N\bar{K}$) channels. In fact, even though the ($N\bar{K}$) channel is closed at the resonance, the ($N\bar{K}$) phase space is bigger than the ($\Sigma\pi$) phase space when $E > 1580$ Mev. On diagonalizing the three channel Born matrix, we obtain for the resultant effective pole strength the values given by a dashed curve in Fig. 3. We also obtain, within about 5% over the range $0 \leq \theta \leq 65^\circ$, the widths $\gamma_{\Sigma\pi} = 0.3 \gamma_{\Lambda\pi}$, or $\Gamma_{\Sigma\pi} = 0.07 \Gamma_{\Lambda\pi}$. The experimental value is $\Gamma_{\Sigma\pi} \lesssim 0.04 \Gamma_{\Lambda\pi}$.

These values, and their insensitivity to θ , can be understood as follows: The wave functions for the $S = -1, T = 1$ components of (3,0) and (2, 2) (as obtained, for example, from table 19d of ref. 21) are:

$$\begin{aligned} \psi(3, 0) &= 6^{-1/2}[(N\bar{K}) + (\Sigma\pi) - (\Xi K)] + \frac{1}{2}[(\Lambda\pi) - (\Sigma\eta)] \\ \psi(2, 2) &= 5^{-1/2}[(N\bar{K}) + (\Xi K)] + (\frac{3}{10})^{1/2}[(\Lambda\pi) + (\Sigma\eta)]. \end{aligned} \quad (35)$$

We observe that it is possible to eliminate *both* the ($\Sigma\eta$) and (ΞK) components by forming the combination

$$6^{1/2}\psi(3, 0) + 5^{1/2}\psi(2, 2) = 2(N\bar{K}) + (\Sigma\pi) + 6^{1/2}(\Lambda\pi). \quad (36)$$

This is a very good approximation to the eigenstate of the truncated Born matrix for $0 \leq \theta \leq 65^\circ$. It actually gives a somewhat smaller value for $\Gamma_{\Sigma\pi}$ than is quoted above.

Only the ($\Xi\pi$) component of the $\Xi_{1/2}^*$ is directly observable. The amplitude of this component is $\frac{1}{2}$, if we assume the $\Xi_{1/2}^*$ is a pure (3, 0) state. Even if we add enough of the (2, 2) configuration to cancel the ($\Xi\eta$) component, this amplitude is only changed to $(\frac{2}{7})^{1/2}$, so it does not much matter which value we use.

The total widths of the $N_{3/2}^*$, Y_1^* , and $\Xi_{1/2}^*$ may be estimated from (32) as follows:

$$\begin{aligned}\Gamma(Y_1^*) &= (11p_{N\pi}^3)^{-1}[p_{\Sigma\pi}^3 + 6p_{\Lambda\pi}^3]\Gamma(N_{3/2}^*) = 0.4\Gamma(N_{3/2}^*) \\ \Gamma(\Xi_{1/2}^*) &= (7p_{N\pi}^3)^{-1}[2p_{\Sigma\pi}^3]\Gamma(N_{3/2}^*) = 0.08\Gamma(N_{3/2}^*).\end{aligned}\quad (37)$$

The available data is uncertain, but is compatible with these ratios.

The mass differences within the $(3, 0)$ multiplet may be estimated by the method of Section III; we shall only describe the results briefly.⁷ If we assume pure $(3, 0)$ states (which is somewhat contrary to the conclusions above) we find that the G-O formula follows from its validity for baryons and pseudoscalar mesons. In this case, the formula implies a constant difference Δ' between M^2 values of successively stranger resonances. One finds, in the notation of Section III:

$$\Delta' = \frac{1}{4}\beta\Delta_2 + \frac{1}{8}\beta\Delta_1 + \frac{1}{8}\gamma\delta_1. \quad (38)$$

We estimate β and γ from (27-30), with the following changes: we now take $\Omega^2 = 3M_B^2$, and make the seemingly plausible assumption that the p^2 dependence of N can be neglected. This gives $\beta \approx 3$ and $\gamma \approx 6$, which leads to a value for Δ' which is twice that observed.

Although the trend of the mass values is correctly reproduced, this numerical discrepancy is rather discouraging, because to do better one would need to examine in some detail how the left hand cuts conspire to produce the effective cutoff of N . Fortunately, as is well known, calculated widths are not subject to these uncertainties.

Figure 3 shows the force in the $(0, 0)$ multiplet, and also the effective attractive force in the $S = -1, T = 0$ state when only $(N\bar{K})$ and $(\Sigma\pi)$ channels are included. These potentials are extremely sensitive functions of the mixing angle, so it is not possible to say anything about the existence of an $S = -1, T = 0, P_{3/2}$ resonance without a better estimate of θ . It is conceivable that the Y_0^* could be such a resonance.

APPENDIX

This appendix has been prepared with the cooperation of Dr. Pekka Tarjanne. It presents some elementary algebraic techniques which we have used to calculate the potentials. The reader is assumed to be familiar with such qualitative features of the irreducible representations (λ, μ) as their hypercharge and isospin content, but a knowledge of the general formal apparatus of continuous group theory or of any special explicit form for the representation matrices is not required. Most of the results derived here, apart from the crossing matrix, have been published elsewhere (14, 21).

⁷ The author was assisted in the calculation by P. Tarjanne.

The group SU_3 is characterized by an eight-component spin vector \mathbf{G} which obeys the commutation relation

$$[G_a, G_b] = -F_{ab}^c G_c. \quad (\text{A.1})$$

In the three-dimensional representation $(1, 0)$ we write $\mathbf{G} = \boldsymbol{\lambda}$. In the eight-dimensional adjoint representation (also often called the regular representation), denoted by $(1, 1)$, we have $G^a \equiv F^a$. Auxiliary matrices D^a are defined by:

$$\{\lambda_a, \lambda_b\} = 2\delta_{ab} + D_{ab}^c \lambda_c. \quad (\text{A.2})$$

The normalization has been fixed so that $\text{Tr } \lambda_a \lambda_b = 3\delta_{ab}$. The equivalence of all three indices of F and D is proved by forming the trace of λ_d times (A.1) and (A.2).

The eigenvalues of the Casimir operator $G^2 = \mathbf{G} \cdot \mathbf{G}$, as calculated by the classical methods (11, 36), are:

$$G^2(\lambda, \mu) = 2(\lambda^2 + \lambda\mu + \mu^2) + 6(\lambda + \mu). \quad (\text{A.3})$$

The particular values of G^2 which we need may also be calculated from properties of the λ , F , and D matrices, as we shall show below; for instance, from (A.2) we have $G^2(1, 0) = \lambda^a \lambda_a = 8$. For each $c \subset a \otimes b$ we define

$$M(c; a, b) = \langle G(a) \cdot G(b) \rangle_c = \frac{1}{2}[G^2(c) - G^2(a) - G^2(b)]. \quad (\text{A.4})$$

Since the trace of any G_a is zero, the average of $M(c)$ over all states vanishes:

$$\sum_c d(c)M(c; a, b) = 0, \quad (\text{A.5})$$

where $d(c)$ is the dimensionality. We may verify that $G^2(1, 1) = 18$, or, in other words, that $\text{Tr } F_a F_b = 18\delta_{ab}$, by applying (A.4) and (A.5) to the decomposition $(1, 0) \otimes (0, 1) = (1, 1) \oplus (0, 0)$.

Substitution of (A.1) and (A.2) into the identities

$$[\lambda_a, \{\lambda_b, \lambda_c\}] \equiv \{\lambda_b, [\lambda_a, \lambda_c]\} + \{[\lambda_a, \lambda_b], \lambda_c\} \quad (\text{A.6})$$

and

$$[\lambda_a, [\lambda_b, \lambda_c]] \equiv \{\lambda_c, \{\lambda_a, \lambda_b\}\} - \{\lambda_b, \{\lambda_a, \lambda_c\}\} \quad (\text{A.7})$$

leads to the following commutators:

$$[F_a, D_b]_{cd} = -F_{ab}^r D_{cdr} \quad (\text{A.8})$$

$$[D_a, D_b]_{cd} = -4(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) - F_{ab}^r F_{cdr}. \quad (\text{A.9})$$

An anticommutation relation is obtained from the expression

$$\{F_a, F_b\}_{cd} = \frac{1}{3} \text{Tr } \{\lambda_d[\lambda_b, [\lambda_a, \lambda_c]] + \lambda_d[\lambda_a, [\lambda_b, \lambda_c]]\}$$

and (A.7):

$$\{F_a, F_b\}_{cd} + \{D_a, D_b\}_{cd} = 2D_{ab}^r D_{ctr} + 8 \delta_{ab} \delta_{cd} - 4(\delta_{ac} \delta_{bd} + \delta_{ad} \delta_{cb}). \quad (\text{A.10})$$

From (A.10) it follows that $\text{Tr } D_a D_b = 10\delta_{ab}$.

Equation (A.8) shows that in the (1, 1) multiplet there is an additional vector \mathbf{D} . Therefore, in the direct product of two representations, the quantities $\bar{M}(c) = \mathbf{D}(a) \cdot \mathbf{D}(b)$ and $W(c) = \mathbf{G}(a) \cdot \mathbf{D}(b)$ are scalars. Since $\text{Tr } D = 0$, one finds in analogy to (A.5), that

$$\sum d(c) \bar{M}(c) = 0, \quad \sum d(c) W(c) = 0.$$

Note that F and D transform differently under the charge-hypercharge reflection R ; one can say that W is a pseudoscalar under R .

We are concerned here with forces between two particles which belong to octets. Since the coupling to these particles of octet mesons is proportional to a linear combination of F and D , the meson exchange potentials will be linear combinations of M , \bar{M} , and W . Graphs B1-3 involve similar combinations of F and D matrices, but with the indices interchanged. We write N and \bar{N} for the modified versions of M and \bar{M} defined by the graphical structure. In a similar way, by interchanging indices in W , three related operators are obtained. From the symmetry properties of the F 's and D 's one finds:

$$N = ME, \quad \text{and} \quad \bar{N} = \bar{M}E, \quad (\text{A.11})$$

where $E_{\alpha\beta} = \delta_{\alpha\beta} \delta_{ab}$ is the operator which interchanges the two interacting particles.

The first step in calculating the values of these operators is to note that we can immediately identify the following as normalized projection operators onto the symmetric and antisymmetric octets and the singlet:

$$\begin{aligned} (P_s)_{\alpha\alpha}^{b\beta} &= 10^{-1} D_{\alpha\alpha} D^{r b\beta} \\ (P_A)_{\alpha\alpha}^{b\beta} &= 18^{-1} F_{\alpha\alpha} F^{r b\beta} \\ (P_0)_{\alpha\alpha}^{b\beta} &= 8^{-1} \delta_{\alpha\alpha} \delta^{b\beta}. \end{aligned} \quad (\text{A.12})$$

The commutator (A.1) gives the relation

$$N = M + 18P_A. \quad (\text{A.13})$$

An expression for \bar{M} is obtained by adding (A.9) and (A.10):

$$\bar{M} = M + 18P_A + 10P_s + 32P_0 - 4. \quad (\text{A.14})$$

We evaluate W^2 by viewing it from the crossed channel:

$$W^2(\text{crossed}) = -M\bar{M}. \quad (\text{A.15})$$

Using (A.14), and crossing back, we obtain:

$$W^2 = 36 + 288P_0 + 90(P_S + P_A) - M^2. \tag{A.16}$$

Since W anticommutes with R , it must vanish in the $(2, 2)$ and $(0, 0)$ multiplets, and have opposite signs in the $(0, 3)$ and $(3, 0)$ multiplets. Within the $(1, 1)$ component, W causes $S \rightarrow A$ transitions. This can be seen most directly by applying W to P_A and P_S , and then using the symmetry properties of F and D to relate the result to M and \bar{M} . We write:

$$(180)^{1/2}Q_{a\alpha}^{b\beta} = F_{ard}D^{rb\beta} + D_{ar\alpha}F^{rb\beta}. \tag{A.17}$$

Then we have:

$$Q^3 = Q = Q^+, \tag{A.18}$$

$$W(1, 1) = (45)^{1/2}Q. \tag{A.19}$$

The sign of W in the tenfold multiplets involves a question of notation. Our convention is that, if in the $\bar{B}HB$ vertex (1) the F and D terms are combined with nearly equal coefficients, it is the $\bar{N}\pi N$ and $\bar{\Xi}\eta\Xi$ couplings that are enhanced, and the $\bar{\Xi}\pi\Xi$ and $\bar{N}\eta N$ couplings that are depressed. In this situation, the potentials in the $2B$ states with hypercharge -2 depend mostly on π exchange, so in the $T = 0$ and $T = 1$ states, which belong, respectively, to the $(0, 3)$ and $(2, 2)$ multiplets, the potentials must have opposite signs. Conversely, the hypercharge $+2$ potentials involve mostly η exchange, so we infer that the $(3, 0)$ and $(2, 2)$ potentials are similar. Therefore, we conclude that $W(3, 0) > 0$, $W(0, 3) < 0$.

Equations (A.11)–(A.19) were used in calculating the potentials in Table I and Eqs. (5) and (6). These identities, when used with (A.4) and (A.5), are also sufficient to determine G^2 for the representations involved.

It may be noted that the discussion given above can be generalized to SU_n , with only trivial modifications arising from the changed dimensionality. In fact, the algebraic structure of a self-consistent SU_n model is rather similar for all $n \geq 3$ (16).

Projection operators for the remaining components of $(1, 1) \otimes (1, 1)$ are obtained by: (1) symmetrizing, (2) subtracting P_A , or P_S and P_0 , (3) distinguishing between $(3, 0)$ and $(0, 3)$ by the sign of W . The normalized operators are:

$$\begin{aligned} P_{(2,2)\alpha\alpha,b\beta} &= \frac{1}{2}(\delta_{ab}\delta_{\alpha\beta} + \delta_{a\beta}\delta_{\alpha b}) - P_{S\alpha\alpha,b\beta} - P_{0\alpha\alpha,b\beta} \\ P_{(3,0)\alpha\alpha,b\beta} &= \frac{1}{2}[\frac{1}{2}(\delta_{ab}\delta_{\alpha\beta} - \delta_{a\beta}\delta_{\alpha b}) - P_{A\alpha\alpha,b\beta}] \\ &\quad + 24^{-1}(F_{ab}^r D_{\alpha\beta r} + D_{ab}^r F_{\alpha\beta r}) \tag{A.20} \\ P_{(0,3)\alpha\alpha,b\beta} &= \frac{1}{2}[\frac{1}{2}(\delta_{ab}\delta_{\alpha\beta} - \delta_{a\beta}\delta_{\alpha b}) - P_{A\alpha\alpha,b\beta}] \\ &\quad - 24^{-1}(F_{ab}^r D_{\alpha\beta r} + D_{ab}^r F_{\alpha\beta r}). \end{aligned}$$

The crossing matrix A_{xy} is defined by

$$P_{x\alpha\beta, b\alpha} = \sum_y A_{xy} P_{y\alpha, b\beta}, \quad (\text{A.21})$$

(for completeness, the P 's in (A.21) must include Q). This matrix determines the contribution of graphs B1-3 when the exchanged baryon state belongs to the representation x . The elements of A , as calculated from (A.12), (A.17), and (A.20), are given in Table II.

ACKNOWLEDGMENTS

The author is grateful to many persons for interesting discussions, especially to Dr. Pekka Tarjanne, who has also checked most of the calculations. The greater part of the work reported here was carried out while the author was a guest at NORDITA (Copenhagen, Denmark), and was partially supported by an NSF Fellowship. The author thanks these organizations for their kindness.

RECEIVED: February 21, 1963

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